HUMAN SCANPATHS ARE NOT LEVY FLIGHTS

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ABSTRACT

It often been proposed that animal foraging patterns follow the pattern of a Levy flight - a random walk process with the step-length drawn from a heavy-tailed distribution, such as a power-law. Recently it has been proposed that human scanpaths might also follow a Levy flight pattern since the eye, which can only fixate a small region at a time, must 'forage' across the scene to extract the most useful information. Using a large corpus of human scanpath data, we show that there is strong evidence that human scanpaths do not follow a Levy flight pattern but instead the simpler gaussian (diffusive) random walk pattern.

***K**eywords* Scanpaths · Levy Flights · Random Walks

1   Introduction

A Levy flight is a random walk process in which the step lengths are drawn from a heavy-tailed distribution, typically a power-law, instead of a more common distribution such as a standard Gaussian. Apart from the step lengths, the steps are made in an isotropic random direction. Because of the heavy-tailed nature of the distribution from which step-lengths are drawn, the Levy flight pattern is one of considerable meandering around a single point, and then a sudden large shift to another point. This contrasts to the more typically Gaussian or diffusive random walk pattern which moves around more slowly and continuously. Example tracks of a Levy flight vs a Gaussian random walk are plotted below:

It has been proposed that the foraging patterns of animals might follow a Levy flight-like pattern. Viswanathan et al. (1996) claimed that the flight times of albatrosses follow a power law, and it has been claimed that the pattern holds for many other species such as jackals (Atkinson et al. (2002)), fruit-flies (Cole, 1995)), and spider monkeys (Ramos-Fernández et al., 2004)). These studies have, however, been criticized on methodological and statistical grounds (Edwards, 2011; Sims et al., 2007), and upon reanalysis of the data, Edwards (2011) claim that simple random walks based on exponential or Gaussian random walks are a better fit. The question of whether ecological foraging or roaming patterns can be accurately modelled as Levy flights is still open.

Moreover, Viswanathan et al. (1999) claim that a Levy flight is the mathematically optimal foraging strategy where food sources are sparse, randomly distributed and depletable (Schlesinger and Klafter, 1986) (but also

20 replenishable). Similarly, Bartumeus et al. (2005) and Viswanathan et al. (2002) claim that Levy flights confer significant advantages over correlated random walks, and other diffusive walk models. Intuitively, this is because the Levy flight tends to stay at one source for a while, and utilise it fully, before striking out until it reaches another source, with minimal backtracking or revisiting recently visited cites (Larralde et al., 1992), and thus the Levy flight also tends to explore more locations over time than the comparable Gaussian random
25 walk (Berkolaiko and Havlin, 1997). However, Benhamou (2007) shows that such patterns need not be Levy flights, but rather a composition of two exponential random walks, one with a significantly higher variance than the other.

Brockmann and Geisel (1999) have proposed that human scanpaths might also follow a Levy flight pattern, in analogy with foraging. This is because human vision, which can only bring a relatively small proportion of the image into foveal
30 focus at a time, must 'forage' across the visual scene to extract the most useful information, which is not generally uniformly distributed. Moreover, human scanpaths can often qualitatively look like Levy flights with a large number of fixations extremely close to each other in succession (microsaccades) before a large saccade moves the fixation point to a completely different location on the image. Boccignone and Ferraro (2004) propose a model in which scanpaths are determined by a Metropolis-Hastings search across a salience map with a Langevin equation whose random term is
35 generated by a Levy-flight power-law distribution. Similarly, Brockmann and Geisel (2000) show that a Levy flight distribution of saccades leads to the quickest convergence of the probability of visiting any image patch, and thus minimizes the amount of time needed to process an image.

Although there are some theoretical results suggesting the utility of Levy flights as scanpaths, to our knowledge there
40 has been little empirical work on whether human saccade magnitudes actually follow such a distribution. In this paper, we analyze a significant corpus of human scanpath data collected under natural viewing conditions, to try to and give this question a firm statistical grounding.

2 Data

The data we used came from the FixaTons dataset (Zanca et al., 2018). Which is comprised of two scanpath datasets,
45 the MIT1003 dataset (Judd et al., 2009) and the Siena12 dataset. The MIT dataset was comprised 1003 images which were observed by 15 subjects, for a total of 15045 scanpaths and 119432 fixations. In the MIT1003 dataset, subjects were located 75cm from the screen and free-viewed the image for 3 seconds.

The Siena12 dataset comprised 12 images which were observed by 23 subjects for a total of 274 scanpaths and 3311
50 fixations. In the Siena12 datasets, subjects were located 72cm from the screen and free-viewed an image for 5 seconds.

The mean number of fixations per scanpath across datasets was 8.02 and the standard deviation about the mean was 2.56. The full distribution of the number of fixations per scanpath can be found in the appendix.This gave us a total of 15319 scanpaths to analyze. We focused on the distribution of distances between fixation points, calculated as simply the
55 euclidean distance between the pixel points where the subjects fixated, since a Levy flight is defined by the distribution of step-lengths - i.e. distances between fixations. The mean pixel-distance between fixations was 139.7 and the standard deviation was 133.4. 107424 distances were analyzed.

3 Method and Results

The defining characteristic of the Levy flight is the heavy tailed distribution of the step-lengths. This distribution is often defined to be a power law, although it could be log-normal or any other heavy tailed distribution.

A power law distribution is a distribution of the form:

$$p(x) = Cx^{-\alpha}$$

where C is a normalization constant and α is the scaling parameter. Typically α ranges between 1 and 3, although this is not necessarily the case.

A power law can often be qualitatively suspected if the histogram frequency of the data forms a straight, negatively sloping line on a log-log plot. This is because, due to the heavy tails, the frequency of unlikely events declines only logarithmically with its distance from the mean. This method, although suggestive, is not statistically sound (Clauset et al., 2009), and a more statistically robust method was used.

A power law distribution was fitted empirically to the data using the maximum likelihood estimator for the scaling parameter α as follows:

$$\hat{\alpha} = 1 + n[\sum_{i=1}^{n} log \frac{x_i}{x_{min}}]^{-1}$$

Where x_min is the minimum cutoff point where the power law distribution begins (the power law distribution tends towards infinity as $\alpha \to 0$).

Alternative distributions such as the Gaussian, the Exponential, and the Log-normal were also fitted to the data in a similar way using their maximum likelihood estimators. These various hypothesis distributions were then compared directly using the likelihood ratio test. The log likelihood ratio [1] was calculated as:

$$R = \sum_{i=1}^{n} [logp(x_i) - logq(x_i)]$$

Where $p(x_i)$ and $q(x_i)$ are the probabilities of each data item under each of the hypothesis distributions. The standard error of the log likelihood ratio can be derived (Clauset et al., 2009) as:

$$\sigma^2 = \frac{1}{n} \sum_{i=1}^{n} [(logp(x_i) - logq(x_i)) - (\mu_p - \mu_q)]$$

where $\mu_p = \frac{1}{n} \sum_i = 1^n logp(x_i)$. From the log likelihood ratio and its standard error, the probability of the ratio being such by chance could be derived, and hence the p-value.

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[1]The log likelihood ratio can be used instead of the likelihood ratio since the logarithm is a continuous function so the ordering of the values will be preserved, and it is often easier to calculate with logs since the likelihood ratio is a product of probabilities which all are < 1, so the final result will become vanishingly small, potentially causing precision issues. With logs the product becomes a sum, so it can be more easily calculated and is more numerically stable.
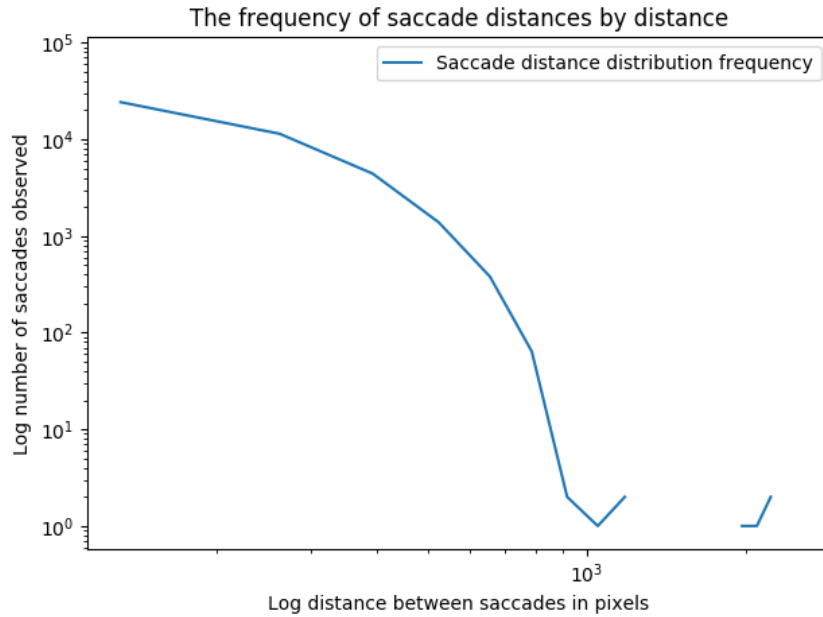
Figure 2: Frequency of saccade differences by distance

First the scanpath distance-between-fixations data was plotted on a histogram of the frequency of the distance against the distance. And then this frequency data was plotted on a log-log chart.
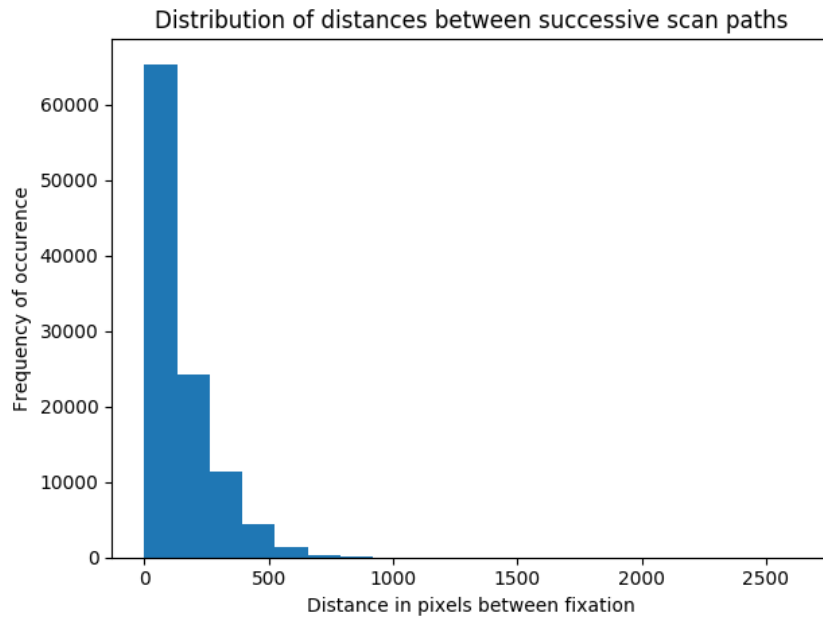

Figure 1: Histogram of distances between successive fixations

This is already not encouraging for the Levy flight hypothesis since if the scanpaths were really derived from a Levy flight, the data should be a straight line on the log-log chart instead of an exponentially decreasing curve.

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Nevertheless, the power-law hypothesis was formally compared using the methodology above with several other distributions including the Gaussian distribution, the exponential distribution and the log-normal distribution. The likelihood ratio for the power-law distribution against the Gaussian distribution was -90854.4 with the p-value of p 0 (90) ($p <<< 0.0001$). This can be confirmed by the graph below which plots the various distributions on the log-log plot, where each distribution has been fitted to the data using the maximum-likelihood estimates of their parameters from the data.
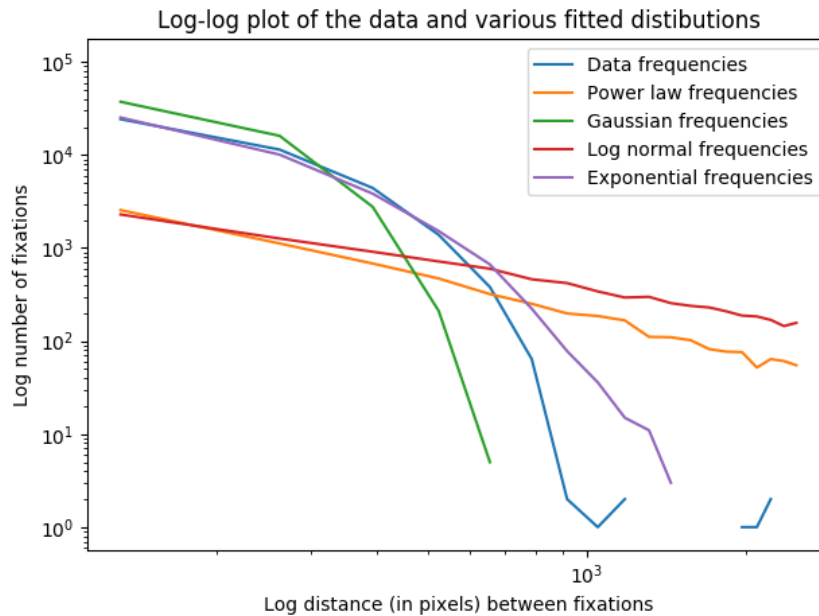
The exponentially decreasing Gaussian and exponential are clearly much better fits than the straight line power-law or log-normal distributions. However, which of the Gaussian or exponential distributions best (95) fits the data? To answer this question, another likelihood ratio test was carried out between the Gaussian and the exponential distribution. The log likelihood ratio was 10927.2 and the p-value was approximately 0 ($p <<< 0.001$).

This demonstrates that the best fit for the scanpath step length data is a Gaussian distribution, and thus that human scanpaths can best be modelled as a simple Gaussian diffusive random walk.

4 Discussion

Although it has been suggested, with reasonable theoretical justification, that human scanpaths might be Levy flights, we have presented strong evidence, based on an analysis of 15319 scanpaths, that this is not the case. Rather, if they are to be modelled as a purely random walk, human scanpaths are much better modelled using a Gaussian rather than any heavy tailed distribution to determine the step-length. Indeed, human fixation patterns across an image are statistically (105) indistinguishable from random Gaussian Brownian motion.

This result is surprising given the theoretical reasons for expecting a Levy flight pattern, and also the prevalence of Levy flights and other super-diffusive walks in the optimal foraging literature, although much of the conclusions of this literature has been questioned and is still under debate. One possibility is that the assumptions of optimal foraging (110) theory and the theoretical models - generally that of sparse and randomly distributed food - are false in the context of visual saliency search. Bartumeus et al. (2002) has shown that when food items are dense, then Levy flights perform no

better than a random Gaussian walk, and it is very possible that the salient regions of the image are generally dense or tightly clustered together. This is especially likely to be the case in the static images typically presented in eye-tracking experiments where the most salient objects are generally presented in the center of the image, and a correspondingly strong center-bias has been observed Tseng et al. (2009).

Secondly, in nature Gaussian distributions are typically observed as a result of the central limit theorem, in which, when many small additive effects combine to produce the final result, that result is distributed as a Gaussian distribution. The fact that scanpath step-lengths follow a Gaussian distribution could be thought of as resulting from a similar process in which the step-length is produced by the additive action of a large number of independent causes in the brain, which combine to cause the ultimate observed step-length. However, it should be cautioned that simply because a process follows some distribution, does not mean that it was generated by that distribution, and that thus the statistical distribution of human scanpaths alone does not necessarily constitute a model of how they are generated.

The result could also have been caused by inadequacies in the dataset. Although 15319 scanpaths were analyzed, there were only about 8 fixations in each scanpath, and this may not have been enough to allow a clear statistical signature of the Levy flight to be observed. Moreover, although the frequency of larger distances decayed exponentially on the log-log plot, instead of linearly as a true Levy flight would, the curve was shifted to the right of that of the fitted Gaussian, thus implying that there was a significantly heavier tail to the distribution than would be expected if the step-distance was purely Gaussian, even if the tail did decay exponentially. Moreover, there were several extremely large distances observed in the scanpath data, which would have vanishingly small probability under the Gaussian distribution of step-lengths and, if these were not artifacts of some kind, then they could be suggestive of heavier tails than were found in this analysis. It is possible that though the majority of the distribution of distances between fixations is Gaussian, the far tails could follow a power-law of some kind, which was not detected here because of insufficient data, as well as insufficient numbers of fixations per scanpath recorded. A more highly-powered study on a much larger corpus of scanpath data would be needed to determine if this is the case. Alternatively, human scanpath data could follow the composite diffusive random walk model of Benhamou (2007) for foraging in which behaviour is determined by two diffusive random walks, one with a significantly higher variance than the other, which can mimic the long-tailed distribution of the Levy flight. This model might actually be more realistic since each Gaussian random walk would correspond to a separate process (macro vs micro-saccades) rather than the Levy flight which would model both by the same mechanism. Future work could test whether the composite model is a better fit of the data than the simple Gaussian model tested here.

5 Conclusion

Overall we have shown that despite claims to the contrary, and theoretical justification from the optimal foraging literature, that human scanpaths do not appear to follow a Levy flight insofar as the distribution of distances between fixations was found not to follow a power-law, or any other heavy-tailed distribution, but rather a standard Gaussian. This result was found on a corpus of 1319 scanpaths from the FixaTons dataset. This means that human scanpaths can be best modelled by a diffusive Gaussian random walk and not a Levy-flight or a super-diffusive walk generated from any heavy-tailed distribution.

6 Appendix

The distribution of the number of fixations per scanpath is shown below:

Histogram of lengths of the scanpaths
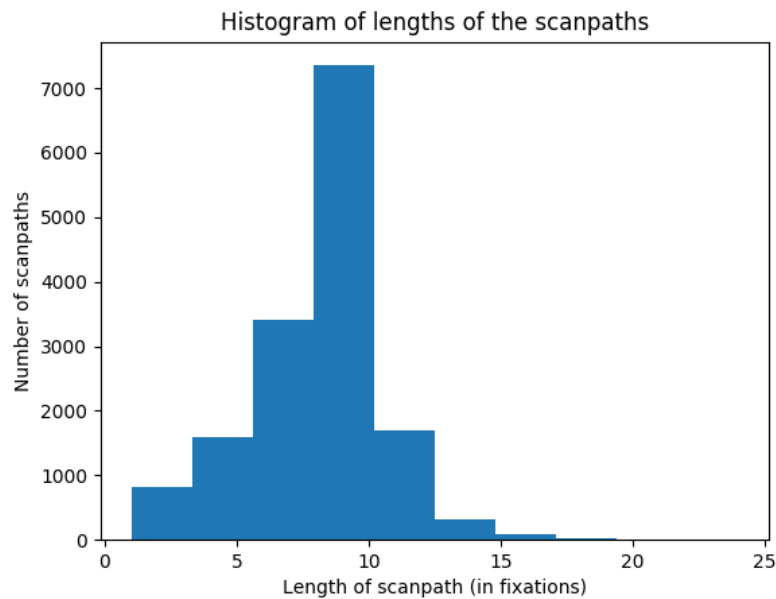
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